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Mathematical technique for transforming the view of a Signal from Time-based to Frequency-based using Fourier Analysis

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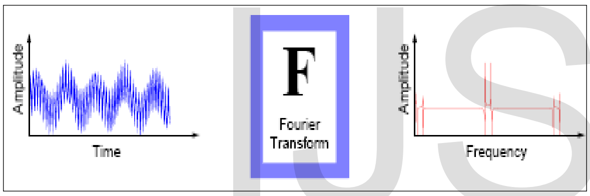
Abstract - Signal analysts already have at their disposal an impressive arsenal of tools. Perhaps the most well-known of these is Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. Another way to think of Fourier analysis is as a mathematical technique for transforming our view of the signal from time-based to frequency-based.

Keywords - STFT, wavelet analysis, Fourier Coefficients, Continuous Wavelet Transform, Wavelets Resolution

INTRODUCTION

Figure 1



For many signals, Fourier analysis is extremely useful because the signal's frequency content is of great importance. So we do need other techniques, like wavelet analysis.

Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place. If the signal properties do not change much over time — that is, if it is what is called a stationary signal—this drawback isn't very important. However, most interesting signals contain numerous non stationary or transitory characteristics: drift, trends, abrupt changes, and beginnings and ends of events. These characteristics are often the most important part of the signal, and Fourier analysis is not suited to detecting them.

Short-Time Fourier Analysis

In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time—a technique called windowing the signal.Gabor's adaptation, called the Short-Time FourierTransform (STFT), maps a signal into a two-dimensional function of time and frequency.

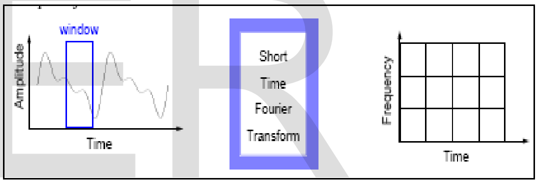


Figure 2

The STFT represents a sort of compromise between the time- and frequency-based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, you can only obtain this information with limited precision, and that precision is determined by the size of the window. While the STFT compromise between time and frequency information can be useful, the drawback is that once you choose a particular size for the time window, that window is the same for all frequencies. Many signals require a more flexible approach—one where we can vary the window size to determine more accurately either time or frequency.

Wavelet Analysis

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information.

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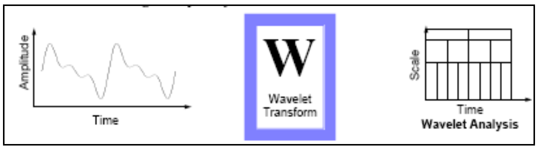

Figure 3

Here's what this looks like in contrast with the time-based, frequency-based, and STFT views of a signal:

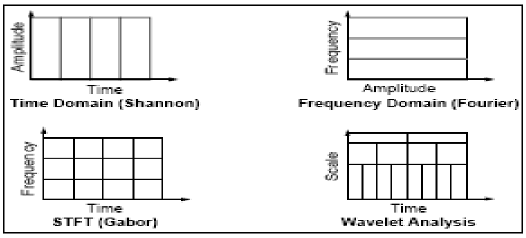

Figure 4

You may have noticed that wavelet analysis does not use a time-frequency region, but rather a time-scale region. For more information about the concept of scale and the link between scale and frequency, see "How to Connect Scale to Frequency?"

EXPERIMENTATION DESIGN AND SETUP

Wavelet Analysis - Advantages

One major advantage afforded by wavelets is the ability to perform local analysis, that is, to analyze a localized area of a larger signal. Consider a sinusoidal signal with a small discontinuity — one so tiny as to be barely visible. Such a signal easily could be generated in the real world, perhaps by a power fluctuation or a noisy switch.

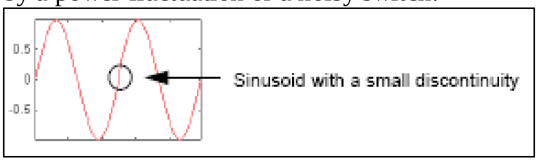

Figure 5

A plot of the Fourier coefficients (as provided by the fft command) of this signal shows nothing particularly interesting: a flat spectrum with two peaks representing a single frequency. However, a plot of wavelet coefficients clearly shows the exact location in time of the discontinuity.

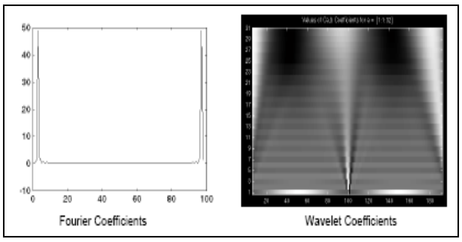

Figure 6

Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss, aspects like trends, breakdown points, discontinuities in higher derivatives, and self-similarity. Furthermore, because it affords a different view of data than those presented by traditional techniques, wavelet analysis can often compress or de-noise a signal without appreciable degradation. Indeed, in their brief history within the signal processing field, wavelets have already proven themselves to be an indispensable addition to the analyst's collection of tools and continue to enjoy a burgeoning popularity today.

What Is Wavelet Analysis?

Now that it has become evident in some situations when wavelet analysis is useful, it is worthwhile asking "What is wavelet analysis?" and even more fundamentally,
"What is a wavelet?" A wavelet is a waveform of effectively limited duration that has an average value of zero.

Compare wavelets with sine waves, which are the basis of Fourier analysis.
Sinusoids do not have limited duration — they extend from minus to plus
infinity. And where sinusoids are smooth and predictable, wavelets tend to be
irregular and asymmetric.

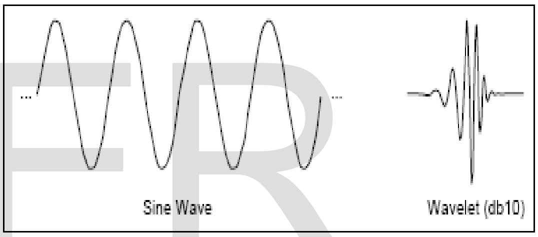

Figure 7

Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or *mother*) wavelet. Just looking at pictures of wavelets and sine waves, you can see intuitively that signals with sharp changes might be better analyzed with an irregular wavelet than with a smooth sinusoid, just as some foods are better handled with a fork than a spoon. It also makes sense that local features can be described better with wavelets that have local extent.

Number of Dimensions

Thus far, the researchers concentrated on only one-dimensional data, which encompasses most ordinary signals. However, wavelet analysis can be applied to two-dimensional data (images) and, in principle, to higher dimensional data. This toolbox uses only one and two-dimensional analysis techniques.

The Continuous Wavelet Transform:

Mathematically, the process of Fourier analysis is represented by the Fourier transform:

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$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

which is the sum over all time of the signal f(t) multiplied by a complex exponential. (Recall that a complex exponential can be broken down into real and imaginary sinusoidal components.) The results of the transform are the Fourier coefficients F(w), which when multiplied by a sinusoid of frequency w yields the constituent sinusoidal components of the original signal. Graphically, the process looks like:

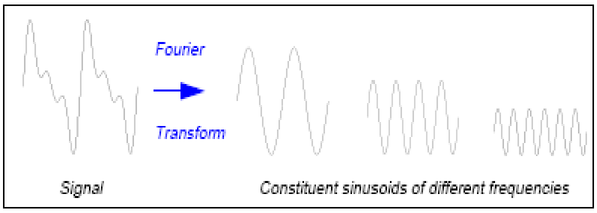

Figure 8

Similarly, the continuous wavelet transform (CWT) is defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function □□□

$$C(scale, position) = \int_{-\infty}^{\infty} f(t)\psi(scale, position, t)dt$$

The result of the CWT is a series many wavelet coefficients C, which are a function of scale and position. Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal:

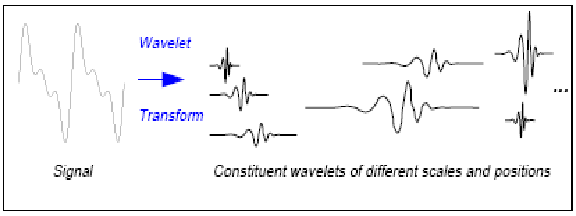

Figure 9

Scaling

We've already alluded to the fact that wavelet analysis produces a time-scale view of a signal and now we're talking about scaling and shifting wavelets.

What exactly do we mean by scale in this context?
Scaling a wavelet simply means stretching (or compressing) it.

To go beyond colloquial descriptions such as "stretching," we introduce the scale factor, often denoted by the letter a.

If we're talking about sinusoids, for example the effect of the scale factor is very easy to see:

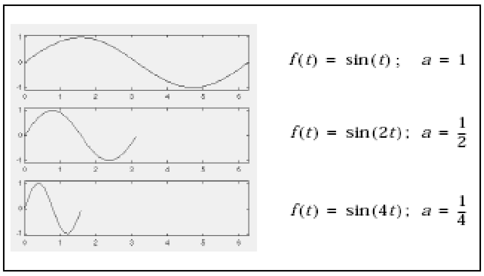

Figure 10

The scale factor works exactly the same with wavelets. The smaller the scale factor, the more "compressed" the wavelet.

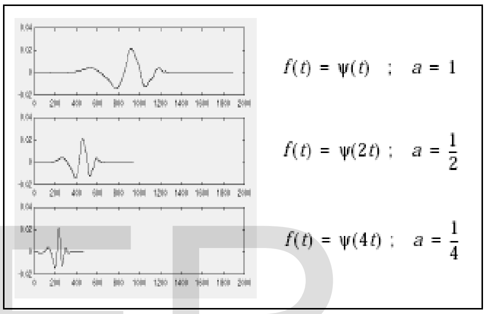

Figure 11

It is clear from the diagrams that for a sinusoid sin(w*t*) the scale factor 'a' is related (inversely) to the radian frequency 'w'. Similarly, with wavelet analysis the scale is related to the frequency of the signal.

Shifting

Shifting a wavelet simply means delaying (or hastening) its onset. Mathematically, delaying a function □□□ (t) by k is represented by □□□ (t – k)

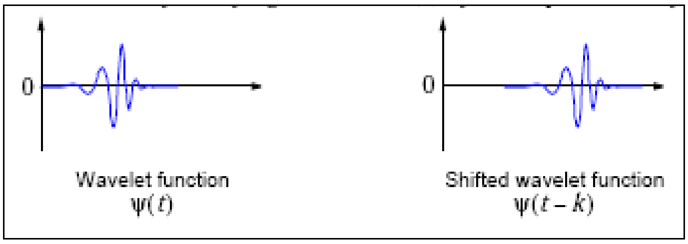

Figure 12

Five Easy Steps to a Continuous Wavelet Transform:

The continuous wavelet transform is the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet. This process produces wavelet coefficients that are a function of scale and position. It's really a very simple process. In fact, here are the five steps of an easy recipe for creating a CWT:

1. Take a wavelet and compare it to a section at the start of the original signal.
2. Calculate a number C that represents how closely correlated the wavelet is with this section of the signal. The higher C is, the more the similarity. More precisely, if the signal energy and the wavelet energy are equal to one, C may be interpreted as a correlation coefficient.

Note that the results will depend on the shape of the wavelet you choose.

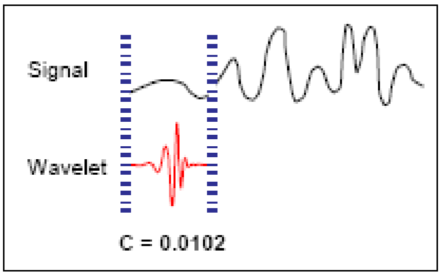

Figure 13

3. Shift the wavelet to the right and repeat steps 1 and 2 until you've covered the whole signal.

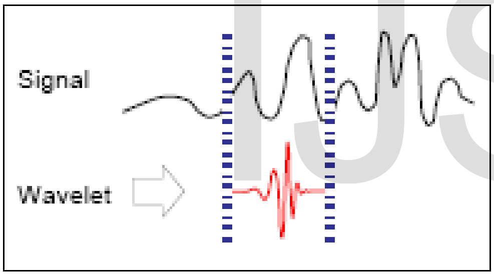

Figure 14

4. Scale (stretch) the wavelet and repeat steps 1 through 3.

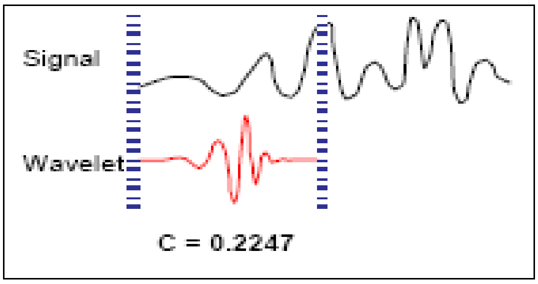

Figure 15

5. Repeat steps 1 through 4 for all scales.

When you're done, you'll have the coefficients produced at different scales by different sections of the signal. The coefficients constitute the results of a regression of the original signal performed on the wavelets.

How to make sense of all these coefficients? You could make a plot on which the *x*-axis represents position along the signal (time), the *y*-axis represents scale, and the color at each *x*-*y* point represents the magnitude of the wavelet coefficient C. These are the coefficient plots generated by the graphical tools.

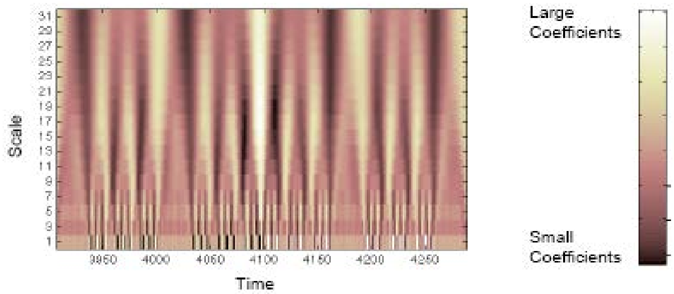

Figure 16

These coefficient plots resemble a bumpy surface viewed from above. If you could look at the same surface from the side, you might see something like this:

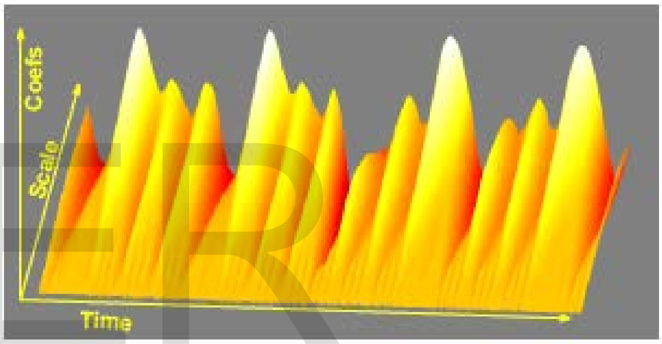

Figure 17

The continuous wavelet transform coefficient plots are precisely the time-scale view of the signal we referred to earlier. It is a different view of signal data than the time- frequency Fourier view, but it is not unrelated.

Scale and Frequency:

Notice that the scales in the coefficients plot (shown as *y*-axis labels) run from 1 to 31. Recall that the higher scales correspond to the most "stretched" wavelets. The more stretched the wavelet, the longer the portion of the signal with which it is being compared, and thus the coarser the signal features being measured by the wavelet coefficients.

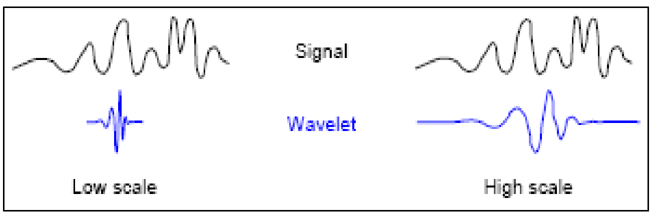

Figure 18

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Thus, there is a correspondence between wavelet scales and frequency as revealed by wavelet analysis:

• Low scale *a*=> Compressed wavelet => Rapidly changing details => High frequency 'w'.

• High scale *a*=>Stretched wavelet=>Slowly changing, coarse features=>Low frequency 'w'.

The Scale of Nature:

It's important to understand the fact that wavelet analysis does not produce a time-frequency view of a signal is not a weakness, but a strength of the technique.

Not only is time-scale a different way to view data, it is a very natural way to view data deriving from a great number of natural phenomena.

Consider a lunar landscape, whose ragged surface (simulated below) is a result of centuries of bombardment by meteorites whose sizes range from gigantic boulders to dust specks.

If we think of this surface in cross-section as a one-dimensional signal, then it is reasonable to think of the signal as having components of different scales—large features carved by the impacts of large meteorites, and finer features abraded by small meteorites.

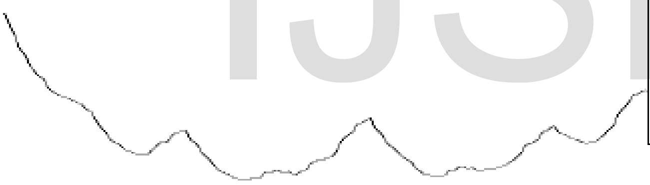

Figure 19

Here is a case where thinking in terms of scale makes much more sense than thinking in terms of frequency. Inspection of the CWT coefficients plot for this signal reveals patterns among scales and shows the signal's possibly fractal nature.

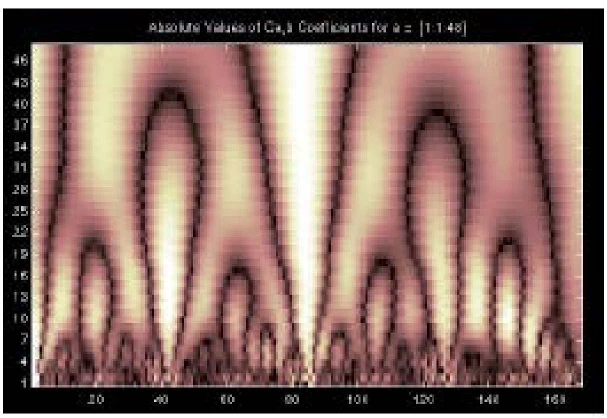

Figure 20

Even though this signal is artificial, many natural phenomena — from the intricate branching of blood vessels and trees, to the jagged surfaces of mountains and fractured metals — lend themselves to an analysis of scale.

What's Continuous About the Continuous Wavelet Transform?

Any signal processing performed on a computer using real-world data must be performed on a discrete signal — that is, on a signal that has been measured at discrete time. So what exactly is "continuous" about it?

What's "continuous" about the CWT, and what distinguishes it from the discrete wavelet transform (to be discussed in the following section), is the set of scales and positions at which it operates.

Unlike the discrete wavelet transform, the CWT can operate at every scale, from that of the original signal up to some maximum scale that you determine by trading off your need for detailed analysis with available computational horsepower.

The CWT is also continuous in terms of shifting during computation, the analyzing wavelet is shifted smoothly over the full domain of the analyzed function.

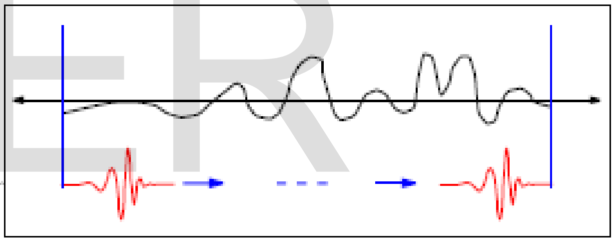

Figure 21

The Discrete Wavelet Transform:

Calculating wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data. What if we choose only a subset of scales and positions at which to make our calculations? It turns out rather remarkably that if we choose scales and positions based on powers of two—so-called dyadic scales and positions—then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the discrete wavelet transform (DWT).

An efficient way to implement this scheme using filters was developed in 1988 by Mallat. The Mallat algorithm is in fact a classical scheme known in the signal processing community as a two-channel sub band coder. This very practical filtering algorithm yields a fast wavelet transform — a box into which a signal passes, and out of which wavelet coefficients quickly emerge. Let's examine this in more depth.

One-Stage Filtering: Approximations and Details:

For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content on the other hand imparts flavor or nuance. Consider the human voice. If you remove the high-frequency components, the voice sounds different but you can still tell what's being said. However, if you remove enough of the low-frequency components, you hear gibberish. In wavelet analysis, we often speak of approximations and details. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components.

The filtering process at its most basic level looks like this:

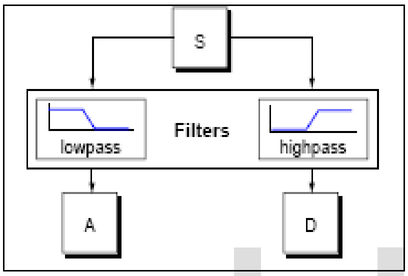

Figure 22

The original signal S passes through two complementary filters and emerges as two signals. Unfortunately, if we actually perform this operation on a real digital signal, we wind up with twice as much data as we started with. Suppose, for instance that the original signal S consists of 1000 samples of data. Then the resulting signals will each have 1000 samples, for a total of 2000.

These signals A and D are interesting, but we get 2000 values instead of the 1000 we had. There exists a more subtle way to perform the decomposition using wavelets. By looking carefully at the computation, we may keep only one point out of two in each of the two 2000-length samples to get the complete information. This is the notion of own sampling. We produce two sequences called cA and cD.

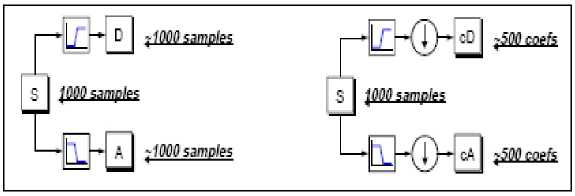

Figure 23

The process on the right which includes down sampling produces DWT Coefficients. To gain a better appreciation of this process let's perform a one-stage discrete wavelet transform of a signal. Our signal will be a pure sinusoid with high-frequency noise added to it.

Here is our schematic diagram with real signals inserted into it:

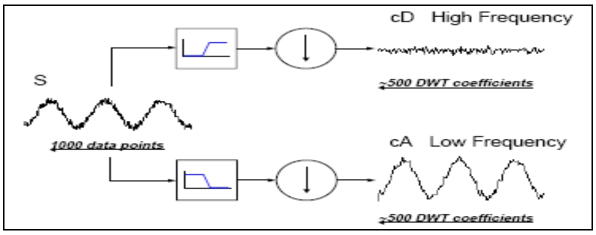

Figure 24

The MATLAB code needed to generate s, cD, and cA is:

```
s = sin(20*linspace(0,pi,1000)) + 0.5*rand(1,1000);
[cA,cD] = dwt(s,'db2');
```

Where db2 is the name of the wavelet we want to use for the analysis.

Notice that the detail coefficients cD is small and consist mainly of a high-frequency noise, while the approximation coefficients cA contains much less noise than does the original signal.

```
[length(cA) length(cD)]
ans = 501 501
```

It may be observed that the actual lengths of the detail and approximation coefficient vectors are slightly *more* than half the length of the original signal. This has to do with the filtering process, which is implemented by convolving the signal with a filter. The convolution "smears" the signal, introducing several extra samples into the result.

Wavelet vs. Fourier analysis:-

In the well-known Fourier analysis, a signal is broken down into constituent sinusoids of different frequencies. These sines and cosines (essentially complex exponentials) are the basis functions and the elements of Fourier synthesis.

Taking the Fourier transform of a signal can be viewed as a rotation in the function space of the signal from the time domain to the frequency domain. Similarly, the wavelet transform can be viewed as transforming the signal from the time domain to the wavelet domain. This new domain contains more complicated basis functions called wavelets, mother wavelets or analyzing wavelets.

Mathematically, the process of Fourier analysis is represented by the Fourier transform:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \qquad (2.1)$$

Which is the sum over all time of the signal f(t) multiplied by a complex exponential.

The results of the transform are the Fourier coefficients F(ω), which when multiplied By a sinusoid of frequency ω, yield the constituent sinusoidal components of the original signal.

A wavelet prototype function at a scale s and a spatial displacement u is defined as:

$$\psi_{s,u}(x) = \sqrt{s}\,\psi\left[\frac{(x-u)}{s}\right] \qquad (2.2)$$

Replacing the complex exponential in Equation 2.1 with this function yields the continuous wavelet transform (CWT):

$$C(s,u) = \int_{-\infty}^{\infty} f(t)\sqrt{s}\,\psi\left[\frac{(x-u)}{s}\right]dt \qquad (2.3)$$

Which is the sum over all time of the signal multiplied by scaled and shifted versions of the wavelet function ψ. The results of the CWT are many wavelet coefficients C, which are a function of scale and position. Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal. The basis functions in both Fourier and wavelet analysis are localized in frequency making mathematical tools such as power spectra (power in a frequency interval) useful at picking out frequencies and calculating power distributions.

The most important difference between these two kinds of transforms is that individual wavelet functions are localised in space. In contrast Fourier sine and cosine functions are non-local and are active for all time t. This localisation feature, along with wavelets localisation of frequency, makes many functions and operators using wavelets. Sparse. When transformed into the wavelet domain. This sparseness, in turn results in a number of useful applications such as data compression, detecting features in images and de-noising signals.

Time-Frequency Resolution:-

A major draw back of Fourier analysis is that in transforming to the frequency domain, the time domain information is lost. When looking at the Fourier transform of a signal, it is impossible to tell when a particular event took place. In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyse only a small section of the signal at a time. A technique called windowing the signal [14]. Gabor.s adaptation, called the Windowed Fourier Transform (WFT) gives information about signals simultaneously in the time domain and in the frequency domain.

To illustrate the time-frequency resolution differences between the Fourier transform and the wavelet transform consider the following figures.

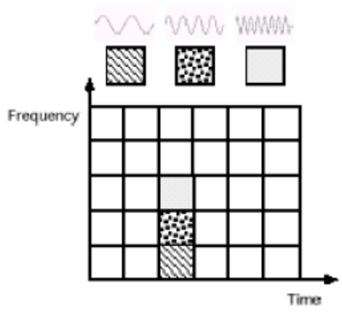

Figure 2.1: WFT Resolution

Figure 2.1 shows a windowed Fourier transform, where the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time frequency plane. An advantage of wavelet transforms is that the windows vary. Wavelet analysis allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones.

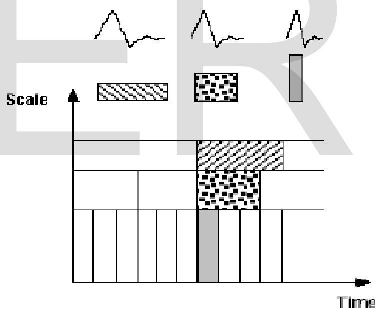

Figure 2.2: Wavelets Resolution

Figure 2.2 shows a time-scale view for wavelet analysis rather than a time frequency region. Scale is inversely related to frequency. A low-scale compressed wavelet with rapidly changing details corresponds to a high frequency. A high-scale stretched wavelet that is slowly changing has a low frequency.

Examples of Wavelets:-

The figure below illustrates four different types of wavelet basis functions.

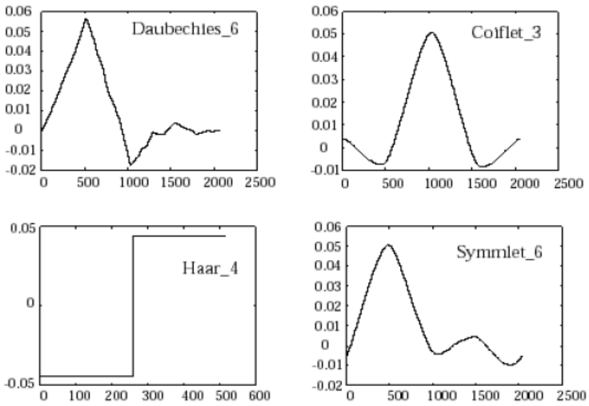

Figure 2.3: Different wavelet families [7]

The different families make trade-offs between how compactly the basis functions are localized in space and how smooth they are. Within each family of wavelets (such as the Daubechies family) are wavelet subclasses distinguished by the number of filter coefficients and the level of iteration. Wavelets are most often classified within a family by the number of vanishing moments. This is an extra set of mathematical relationships for the coefficients that must be satisfied. The extent of compactness of signals depends on the number of vanishing moments of the wavelet function used.

The Discrete Wavelet Transform:-

The Discrete Wavelet Transform (DWT) involves choosing scales and positions based on powers of two. So called dyadic scales and positions. The mother wavelet is rescaled or dilated by powers of two and translated by integers.

Specifically, a function f(t) $\in$ L2(R) (defines space of square integrable functions) can be represented as

$$f(t) = \sum_{j=1}^{L} \sum_{k=-\infty}^{\infty} d(j,k)\psi(2^{-j}t-k) + \sum_{k=-\infty}^{\infty} a(L,k)\phi(2^{-L}t-k) \quad (2.4)$$

The function ψ(t) is known as the mother wavelet, while φ(t) is known as the scaling Function. The set of functions

$$\{\sqrt{2^{-l}}\phi(2^{-L}t-k), \sqrt{2^{-j}}\psi(2^{-j}t-k) \mid j \le L, j,k,L \in Z\}$$

Where Z is the set of integers is an orthonormal basis for L2(R).

The numbers a(L, k) are known as the approximation coefficients at scale L, while d(j,k) are known as the detail coefficients at scale j.

The approximation and detail coefficients can be expressed as:

$$a(L,k) = \frac{1}{\sqrt{2^L}} \int_{-\infty}^{\infty} f(t)\phi(2^{-L}t-k)dt \quad (2.5)$$

$$d(j,k) = \frac{1}{\sqrt{2^j}} \int_{-\infty}^{\infty} f(t)\psi(2^{-j}t-k)dt \quad (2.6)$$

To provide some understanding of the above coefficients consider a projection fl(t) of the function f(t) that provides the best approximation (in the sense of minimum error energy) to f(t) at a scale l. This projection can be constructed from the coefficients a(L, k), using the equation

$$f_l(t) = \sum_{k=-\infty}^{\infty} a(l,k)\phi(2^{-l}t-k).$$

As the scale l decreases, the approximation becomes finer, converging to f(t) as l → 0. The difference between the approximation at scale l + 1 and that at l, fl+1(t) - fl(t), is completely described by the coefficients d(j, k) using the equation

$$f_{l+1}(t) - f_l(t) = \sum_{k=-\infty}^{\infty} d(l,k)\psi(2^{-l}t-k).$$

Using these relations, given a(L, k) and {d(j, k) | j ≤ L}, it is clear that we can build the approximation at any scale. Hence, the wavelet transform breaks the signal up into a coarse approximation fL(t) (given a(L, k)) and a number of layers of detail {fj+1(t)-fj(t)| j < L} (given by {d(j, k) | j ≤ L}). As each layer of detail is added, the approximation at the next finer scale is achieved.

Vanishing Moments

The number of vanishing moments of a wavelet indicates the smoothness of the wavelet function as well as the flatness of the frequency response of the wavelet filters (filters used to compute the DWT).Typically a wavelet with p vanishing moments satisfies the following equation .

$$\int_{-\infty}^{\infty} t^m \psi(t)dt = 0 \qquad \text{for m = 0,…., p-1,}$$

or equivalently,

$$\sum_k (-1)^k k^m c(k) = 0 \quad \text{for m = 0,…., p-1.}$$

For the representation of smooth signals, a higher number of vanishing moments leads to a faster decay rate of wavelet coefficients. Thus, wavelets with a high number of vanishing moments lead to a more compact signal representation and are hence useful in coding applications.

However, in general, the length of the filters increases with the number of vanishing moments and the complexity of computing the DWT coefficients increases with the size of the wavelet filters.

RESULTS AND DISCUSSION

The Discrete Wavelet Transform (DWT) coefficients can be computed by using Mallat.s Fast Wavelet Transform algorithm. This algorithm is sometimes referred to as the two-channel sub-band coder and involves filtering the input signal based on the wavelet function used.

Implementation Using Filters

To explain the implementation of the Fast Wavelet Transform algorithm consider the following equations:

$$\phi(t) = \sum_k c(k)\phi(2t-k) \qquad (2.7)$$

$$\psi(t) = \sum_k (-1)^k c(1-k)\phi(2t-k) \qquad (2.8)$$

$$\sum_k c_k c_{k-2m} = 2\delta_{0,m} \qquad (2.9)$$

The first equation is known as the twin-scale relation (or the dilation equation) and defines the scaling function φ. The next equation expresses the wavelet ψ in terms of the scaling function φ. The third equation is the condition required for the wavelet to be
Orthogonal to the scaling function and its translates.

The coefficients c(k) or {c0, .., c2N-1} in the above equations represent the impulse response coefficients for a low pass filter of length 2N, with a sum of 1 and a norm of1/2
The high pass filter is obtained from the low pass filter using the relationship
g ( )k c(k) k = −1 1− , where k varies over the range (1 . (2N . 1)) to 1.

Equation 2.7 shows that the scaling function is essentially a low pass filter and is used to define the approximations. The wavelet function defined by equation 2.8 is a high pass filter and defines the details.

Starting with a discrete input signal vector s, the first stage of the FWT algorithm decomposes the signal into two sets of coefficients. These are the approximation coefficients cA1 (low frequency information) and the detail coefficients cD1 (high frequency information), as shown in the figure below.

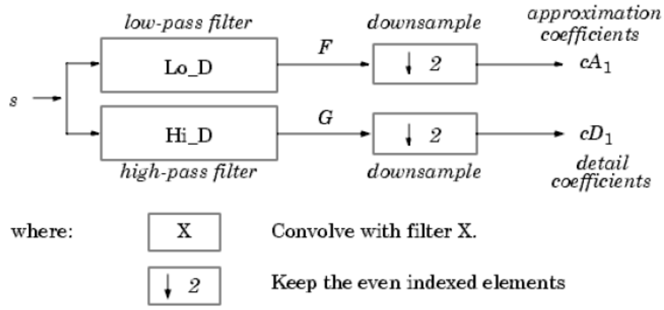

Figure 2.4: Filtering operation of the DWT [15]

The coefficient vectors are obtained by convolving s with the low-pass filter Lo_D for Approximation and with the high-pass filter Hi_D for details. This filtering operation is then followed by dyadic decimation or down sampling by a factor of 2. Mathematically the two-channel filtering of the discrete signal s is represented by the expressions:

$$cA_1 = \sum_k c_k s_{2i-k}\ , \qquad cD_1 = \sum_k g_k s_{2i-k} \qquad (2.10)$$

These equations implement a convolution plus down sampling by a factor 2 and give the forward fast wavelet transform.

If the length of each filter is equal to 2N and the length of the original signal s is equal to n, then the corresponding lengths of the coefficients cA1 and cD1 are given by the formula:

$$\text{floor}\left(\frac{n-1}{2}\right) + N \qquad (2.11)$$

This shows that the total length of the wavelet coefficients is always slightly greater than the length of the original signal due to the filtering process used.

Multilevel Decomposition:-

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution Components. This is called the wavelet decomposition tree.

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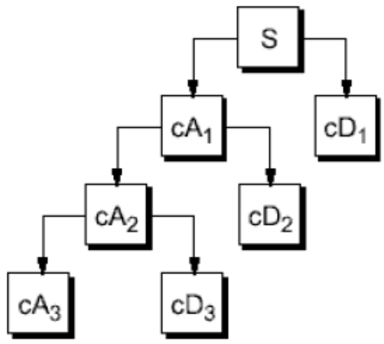

Figure 2.5: Decomposition of DWT coefficients

The wavelet decomposition of the signal s analysed at level j has the following structure [cAj, cDj, ..., cD1].

Looking at a signals wavelet decomposition tree can reveal valuable information. The diagram below shows the wavelet decomposition to level 3 of a sample signal S.

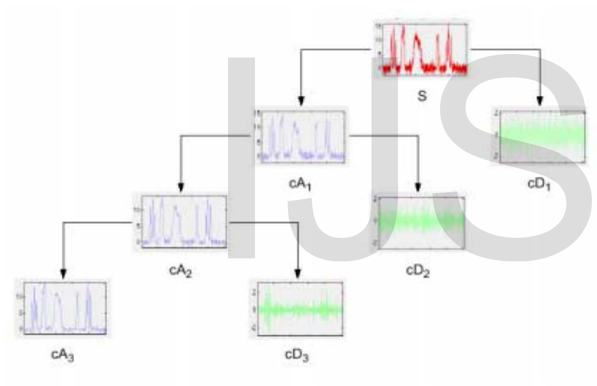

Figure 2.6: Level 3 Decomposition of Sample Signal S [15]

Since the analysis process is iterative, in theory it can be continued indefinitely. In reality, the decomposition can only proceed until the vector consists of a single sample. Normally, however there is little or no advantage gained in decomposing a signal beyond a certain level. The selection of the optimal decomposition level in the hierarchy depends on the nature of the signal being analyzed or some other suitable criterion, such as the low-pass filter cut-off.

Signal Reconstruction:-

The original signal can be reconstructed or synthesized using the inverse discrete wavelet transform (IDWT). The synthesis starts with the approximation and detail coefficients cAj and cDj, and then reconstructs cAj-1 by up sampling and filtering with the reconstruction filters.

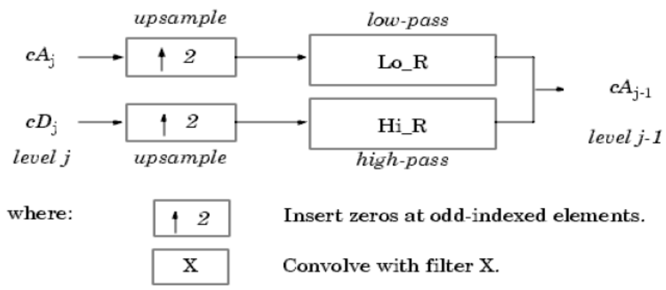

Figure 2.7: Wavelets Reconstruction [15]

The reconstruction filters are designed in such a way to cancel out the effects of aliasing introduced in the wavelet decomposition phase. The reconstruction filters (Lo_R and Hi_R) together with the low and high pass decomposition filters, forms a system known as quadrature mirror filters (QMF).

For a multilevel analysis, the reconstruction process can itself be iterated producing successive approximations at finer resolutions and finally synthesizing the original signal.

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